A CAD-Suitable Approach for the Analysis of Nonuniform MMIC and MHMIC Transmission Lines

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Abstract— **A new method of moment-based formulation for the solution of the telegraphist's equations in nonuniform transmission lines is presented. Entire domain basis functions that build in a frequency variation are used to cover wider frequency and physical dimension ranges. The results obtained using the proposed formulation are validated by comparison to those obtained by a CAD package and to measured data. Different nonuniform lines in microstrip and coplanar technologies on monolithic microwave/millimeter wave integrated circuit (MMIC) and miniaturized hybrid microwave integrated circuit (MHMIC) substrates are investigated with an application to the design a matched taper transition in a MMIC coplanar line.**

I. Introduction

Over the past several years considerable work has been carried out in the area of microstrip discontinuity modeling. More recently, CAD-suitable discontinuity models for coplanar transmission lines have been derived [1]. As a result, a fair amount of CAD-oriented models of various discontinuities is now available. However, by contrast, little work has been done in the area of nonuniform transmission lines modeling except for costly fully three-dimensional (3-D) field simulators, the numerical approach described in [2] and [3], or the analytical solution for single-line exponential tapers (see for example [4]). Consequently, CAD models for nonuniform transmission lines are very limited and even the few that exist, such as for example the linearly tapered microstrip line, have limitations, i.e., the ratio of the taper's length to the difference in width must be smaller than 0.6 [5]. In the case nonuniform coplanar transitions, no CAD models are available at all. In many cases, one is therefore left with a tedious and time-consuming cascading approach as the only option.

In this paper, a new formulation of the nonuniform transmission line problem is presented. The proposed approach is applicable to any guiding structure whose fundamental mode can be treated as a quasi-TEM mode. In particular, a number of nonuniform microstrip and coplanar waveguide transitions will be analyzed to illustrate the method. The accuracy of the proposed technique is validated by comparison with the results of a CAD package, using built-in models where applicable and cascading otherwise, and with measured data.

II. Formulation

A schematic representation of a nonuniform section of a quasi-TEM supporting transmission line is shown in Fig. 1. For the fundamental mode, the propagation in such a structure can be described in terms of the telegraphers' equations with frequency and position dependent line parameters, namely

$$\begin{cases} \frac{\partial I(f,z)}{\partial z} = -Y(f,z)V(z) \\ \frac{\partial V(f,z)}{\partial z} = -Z(f,z)I(z) \end{cases} \tag{1}$$

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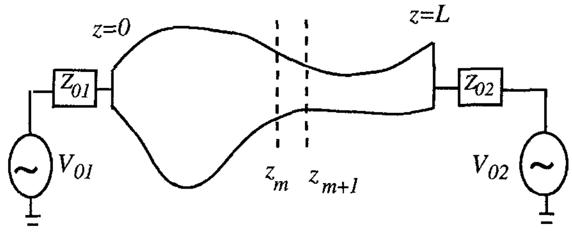

Fig. 1. Schematic representation of the geometry of a nonuniform line.

where f is the frequency and Z and Y are the per-unit length impedance and admittance of the line, respectively. These parameters are assumed to be known from the line geometry at a given z-position and from the frequency. For microstrip lines, the accurate closed form expressions in [6], including dispersion effects, are used while for coplanar lines the expressions in [7] and [8] are employed. For other guiding structures, where no closed form expressions are available, these parameters can be computed by one of a number of different numerical techniques with varying degrees of accuracy and computational cost. It is important to note that these parameters are in general complex of the form

$$Z(z) = R(z) + j\omega L(z) \quad \text{and} \quad Y(z) = G(z) + j\omega C(z). \tag{2}$$

This allows us to include losses due to finite conductor conductivity and thickness through the resistance term, as well as those due to dielectrics through the conductance term. The per-unit length parameters *RLCG* can be computed from the basic transmission line's constants [6] (characteristic impedance: Z_0, phase constant: β, attenuation constant due to conductor loss: α_c, attenuation constant due to dielectric loss: α_d) at a given z position as follows:

$$\begin{cases} L = \frac{Z_0\beta}{2\pi f} & R = Z_0(\alpha_c + \alpha_d) \\ C = \frac{\beta}{Z_0 2\pi f} & G = \frac{(\alpha_c + \alpha_d)}{Z_0} \end{cases}. \tag{3}$$

Next, we proceed to formulate a method of moment solution of the coupled equations in (1). The key to such a solution is the accurate representation of the unknown current and voltage along the line. First, we note that a conventional subdomain basis functions (partially overlapping) expansion approach would not work here. This is due to the discontinuity that would result in either the current or voltage as a consequence of the derivative with respect to z and the coupling of (1). Therefore, an entire-domain basis functions formulation is needed. In the present approach, we propose to use frequency-varying basis functions by expanding the currents and voltages in terms of forward and backward propagating waves with different wavenumbers such that

$$\begin{cases} I(z) = \sum_{i=1}^{N} a_i e^{-\gamma_i z} + b_i e^{\gamma_i z} = \sum_{i=1}^{N} a_i F_i(z) + b_i B_i(z) \\ V(z) = \sum_{i=1}^{N} c_i e^{-\gamma_i z} + d_i e^{\gamma_i z} = \sum_{i=1}^{N} c_i F_i(z) + d_i B_i(z) \end{cases} \tag{4}$$

where $\{a_i, b_i, c_i, d_i\}$ are unknown coefficients and where the frequency-dependence is built in the propagation constants $\gamma_i = \alpha_i + j\beta_i$. Note that the expansion in (4) is not a spectral representation since the set of propagation constants $\{\gamma_i\}$ is not related to the spatial coordinate z and the line length L, but rather to the line's cross-sectional dimensions at a set of points $\{z_i\}$ along the line and to frequency. Note also that, in the limiting case of a uniform line, only one set of basis functions, namely F_1 and B_1, will suffice to solve the problem exactly for all frequencies.

Substituting (4) into (1) and testing with F_n and B_n, we obtain a matrix equation whose entries are given in terms of the following inner products:

$$\langle F_i, F_n\rangle\langle F_i, B_n\rangle\langle B_i, F_n\rangle\langle B_i, B_n\rangle \quad (5a)$$
$$\langle Z(z)F_i, F_n\rangle\langle Z(z)F_i, B_n\rangle\langle Z(z)B_i, F_n\rangle\langle Z(z)B_i, B_n\rangle \quad (5b)$$
$$\langle Y(z)F_i, F_n\rangle\langle Y(z)F_i, B_n\rangle\langle Y(z)B_i, F_n\rangle\langle Y(z)B_i, B_n\rangle \quad (5c)$$

where the inner product definition used is

$$\langle f, g\rangle = \int_0^L f(z)g(z)dz. \quad (6)$$

The terms of (5a) are easily evaluated in closed form. However, to evaluate the terms of (5b) and (5c) which involve the position-varying line parameters, a slightly different procedure is followed. First, the total line length is subdivided in to M equal segments (see Fig. 1). Then, the per-unit length parameters $Y(f, z)$ and $Z(f, z)$ are represented by a piece-wise linear function such that, at the given frequency f and over the mth segment we have

$$\begin{cases} Y(z) = A_{ym}z + B_{ym} \\ Z(z) = A_{zm}z + B_{zm} \end{cases} \quad \text{for } z_m \leq z \leq z_{m+1} \quad (7)$$

where A_{ym}, A_{zm}, B_{ym} and B_{zm} are computed from $Y(z_m)$, $Y(z_{m+1})$, $Z(z_m)$ and $Z(z_{m+1})$. The integrals of (5b) and (5c) can then be written as sums of the general form

$$\langle X(z)f, g\rangle = \sum_{m=1}^{M} A_{xm}\int_{z_m}^{z_{m+1}} z\cdot f(z)g(z)dz + B_{xm}\int_{z_m}^{z_{m+1}} f(z)g(z)dz \quad (8)$$

where $f(z)$ and $g(z)$ represent combinations of the functions F_i and B_i. Consequently, closed form expressions for these integrals are easily obtained in terms of the known $A_{.m}$ and $B_{.m}$ coefficients at each frequency.

It should be noted that the integrals in (5) involving $F_{i,n}$ and $B_{n,i}$ have denominators of the form $(\gamma_{i,n} - \gamma_{n,i})$. For the case of $\gamma_i = \gamma_n$, which arises when $i = n$ or when the same geometric parameters of the line are repeated at different positions, we can show that these integrals have a finite values which can be used in the numerical computations. However, due to the finite precision of computers' arithmetic, care must be taken for the cases when $i \neq n$ and $\gamma_i \cong \gamma_n$ to avoid overflow errors. This can accomplished by choosing a distinct set of γ_i through a linear interpolation between the minimum and maximum values of γ associated with the structure being considered; or by proper choice of a tolerance criterion on $|\gamma_i - \gamma_n|$ for which the integrals are replaced by their corresponding finite values.

With the method of moments matrix thus filled, boundary conditions are applied to complete the system of equations and compute the scattering parameters of the nonuniform line. This is done by considering the terminal conditions shown in Fig. 1 and which give

$$\begin{cases} V(0) = V_{01} - Z_{01}I(0) \\ V(L) = V_{02} + Z_{02}I(L) \end{cases} \quad (9)$$

where $Z_{01} = Z_{02} = 50\ \Omega$. To obtain the scattering parameters of the line, we solve the problem twice: once with

$$\left[{}^{V_{01}=1}_{V_{02}=0}\right]$$

and a second time with

$$\left[{}^{V_{01}=0}_{V_{02}=1}\right].$$

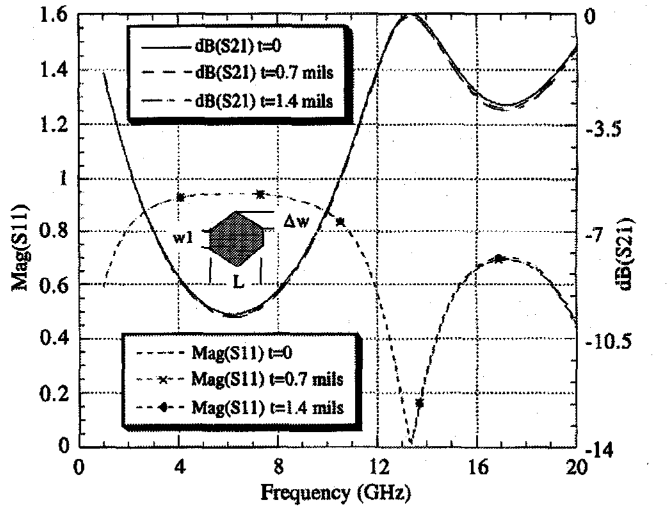

Fig. 2. Scattering parameters of an end-to-end taper as a function of conductor thickness. $w1 = 0.75$ mm, $\Delta w = 3.5$ mm, $L = 5$ mm, $\varepsilon_r = 10$, $h = 0.254$ mm, $\tan\delta = 0.0001$, conductor $\sigma = 5.8 \times 10^7$ S/m.

Using these results, and the S-parameters definition

$$S_{ij} = \frac{2V_{ij} - V_{0j}}{V_{0i}}$$

where V_{ij} is the voltage at port i when port j is excited and V_{0i} is the excitation voltage at port i ($V_{0j} = 0$ for $i \neq j$), we obtain the four S-parameters of the line.

III. Results

The above approach has been implemented and tested on a number of structures using only a moderate number of basis functions (between 3 and 5) with good results. A wide range of linear

$$w(z) = w1 + 2\Delta w(1 - \frac{|z - L/2|}{L/2})$$

microstrip tapers and sinusoidally-modulated periodic microstrip structures

$$w(z) = w_0(1 - m\cos(\frac{2\pi z}{L}))$$

were investigated and compared to MDS models [5] and to measured results with good agreement [9]. Here, the effects of the conductor's thickness and finite conductivity, as well as those of dielectric loss, on the unit-cell of Fig. 2 in reference [9] are studied and the results for different strip thicknesses are shown in Fig. 2.

In addition to the microstrip transitions, tapers in ground-backed coplanar waveguide (CPWG) on a MMIC substrate and coplanar waveguide without a lower ground plane (CPW) on a MHMIC substrate were analyzed. Since no CAD model exists for these transitions, cascaded sections were used in MDS to simulate the tapers. The S-parameters of both tapers resulting from the present approach and the MDS simulations are shown in Figs. 3 and 4 for CPWG and CPW, respectively. It is seen that as the number of cascaded sections increases, the MDS results converge toward the results of the present approach. In order to validate these results with measurement data, special unit cells made of end-to-end tapers are needed to accommodate the fixed pitch of the probes available to us. One such cell was fabricated on the MHMIC substrate and measured with a probe station and the HP8510 Network Analyzer. The results of the measurements are presented in Fig. 5. Excellent agreement

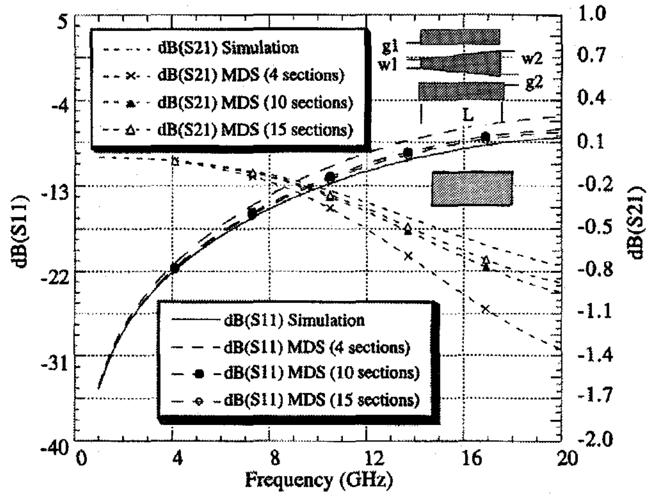

Fig. 3. Scattering parameters of a linear CPWG taper on a MMIC substrate. $w1 = 0.2$ mm, $g1 = 0.6$ mm, $w2 = 0.8$ mm, $g2 = 0.1$ mm, $L = 2$ mm, $h = 0.635$ mm, $\varepsilon_r = 12.9, Z1 = 65.43\ \Omega, Z2 = 27.48\ \Omega$.

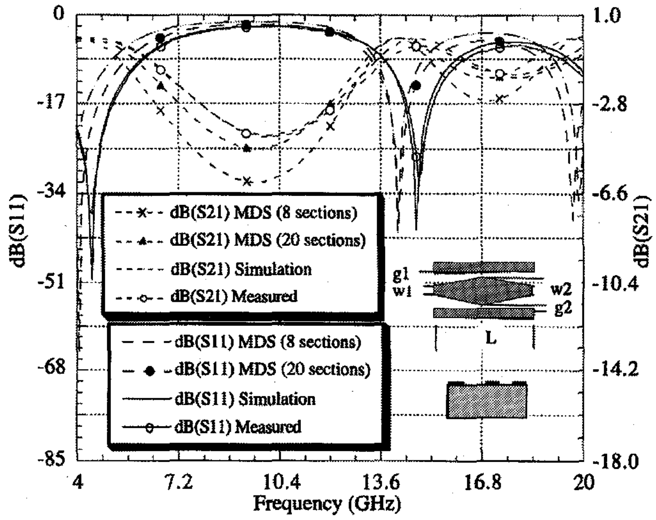

Fig. 5. Computed and measured scattering parameters of an end-to-end CPW taper. $w1 = 0.1$ mm, $g1 = 0.6$ mm, $w2 = 0.8$ mm, $g2 = 0.1$ mm, $L = 10$ mm, $h = 0.254$ mm, $\varepsilon_r = 10.0$.

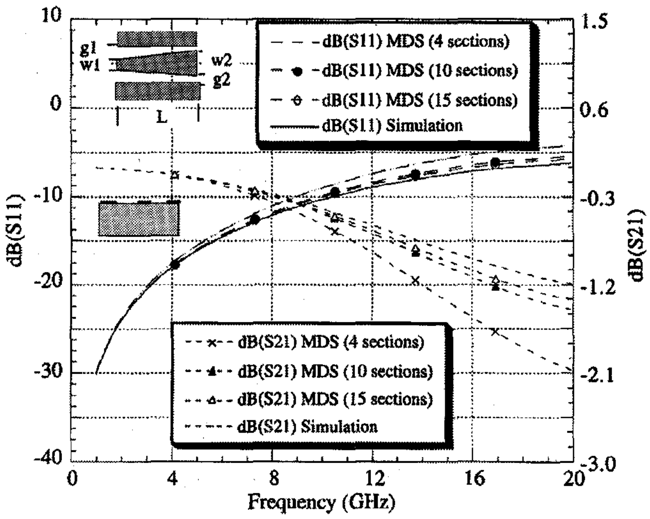

Fig. 4. Scattering parameters of a linear CPW taper on a MHMIC substrate. $w1 = 0.1$ mm, $g1 = 0.6$ mm, $w2 = 0.8$ mm, $g2 = 0.1$ mm, $L = 2.5$ mm, $h = 0.254$ mm, $\varepsilon_r = 10.0, Z1 = 118.08\ \Omega, Z2 = 40.27\ \Omega$.

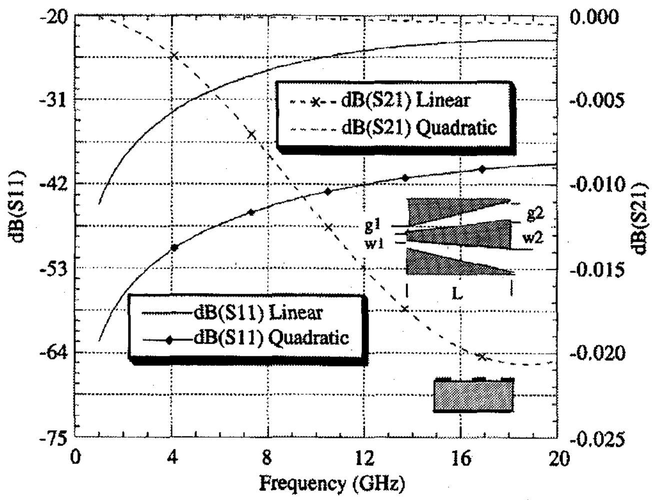

Fig. 6. Scattering parameters of matched taper transition in CPWG. $w1 = 0.138$ mm, $g1 = 0.1$ mm, $w2 = 0.414$ mm, $g2 = 0.6$ mm, $L = 2$ mm, $h = 0.635$ mm, $\varepsilon_r = 12.9, Z1 = Z2 = 50\ \Omega$.

between simulation and measurements is seen which demonstrates the accuracy and validity of the proposed approach.

Finally, an interesting application of the present approach is in the design and analysis of matched transitions. Unlike the microstrip line, where substrate height must be varied if one is to obtain a matched taper transition [10], a combination of central conductor width and gap spacing variations make a matched taper transition much easier to accomplish in coplanar technology. For example, a 50 Ω impedance can be obtained with ($w = 0.138$ mm, $g = 0.1$ mm) or ($w = 0.414$ mm, $g = 0.6$ mm). Intuitively, a matched transition would be obtained with a linear taper in both w and g. However, given that the relationship between the impedance and w and g is not quite linear, a more precisely matched transition can be obtained with a linear taper in g and a quadratic taper in w. For the transition considered here, and for which the results are shown in Fig. 6, the quadratic transition is given by the equation: $w(z) = 0.14 + 0.238z - 52.45z^2$, where w and z are in mm. This profile was obtained as follows: at each position along z, $g(z)$ is computed by linear interpolation between $g(0)$ and $g(L)$. With $g(z)$ known and fixed, $w(z)$ is obtained by synthesizing a 50 Ω line. A curve fitting of the resulting set of points is then performed to determine the profile for $w(z)$. It was found that a quadratic polynomial yielded good fit of the profile of $w(z)$.

IV. Conclusion

A new formulation using a method of moments approach with frequency-varying basis functions for the simulation of nonuniform transmission lines has been presented. The accuracy of the proposed technique was tested by comparison to existing empirical models and to measured data. The effects of finite conductor thickness and conductivity have also been included and tested. Furthermore, the application of this technique to transitions in coplanar waveguides was demonstrated through the analysis of linear tapers, matched quadratic tapers, as well as the analysis and measurement of cells made of end-to-end tapers. The fact that the basis functions used in the proposed approach build in the frequency dependence of the current and voltage makes it possible to solve complicated structures with only a small number of basis functions making the method quite efficient. The speed and efficiency of the algorithm used for the numerical implementation of the method make it particularly attractive and practical for CAD applications to simulate transitions in a wide range of transmission lines. The main limitation of the present approach is the relatively smooth variation of the transmission line parameters (RLCG) required (i.e., sharp discontinuity effects cannot be modeled at this stage). Finally, with such an efficient technique, circuit synthesis problems using nonuniform lines for a variety of applications are currently undertaken.

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